Discrete Variable Approximation to Minimum Weight Panels with Fixed Flutter Speed

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Theme

THIS paper presents a numerical technique for determining the minimum-weight design of a one-dimensional panel for which an aeroelastic eigenvalue characterizing the flutter speed is held constant. The governing differential equations are approximated by sets of difference equations adjoined to the weight function via a penalty function. A conjugate gradient method is applied to the resulting sequence of unconstrained minimization problems. Numerical results are obtained for solid simply-supported panels with constant inplane stresses. These results supplement those of Armand and Vitte[1] who posed a solid panel problem without inplane stresses and Weisshaar[2] who obtained numerical solutions for a sandwich panel without inplane stresses using a minimum thickness constraint. The discrete variable technique has been applied previously to flight path optimization[3] and is thought to have several advantages, including 1) ease of implementation, 2) exact satisfaction of boundary conditions, 3) ability to treat the frequency parameter α as an additional problem variable, 4) ability to avoid the differential equation end point singularities without imposing a minimum thickness constraint, and 5) ease in obtaining adequate initial solution estimates.

Contents

Consider an initially flat solid panel of infinite span and length a. One side of the simply supported panel is exposed to a parallel, high Mach number, supersonic flow. Assuming linear elastic bending and linearized static aerodynamic strip theory,[4] the nondimensional differential equation of equilibrium is

$$[t^3(x)w'']'' + R_x w'' + \lambda_o w' - (\alpha\pi)^4 t(x)w = 0 \qquad (1)$$

where $x = X/a$, X = distance along panel; $w = W/a$, W = panel deflection; $t(x) = T(x)/T_o$, T = panel thickness, T_o = thickness of reference panel; $R_x = N_x a^2/D_o$, N_x = constant inplane stress (positive for compression); D_o = stiffness of reference panel; $\alpha = (\omega/\omega_f)^{1/2}$, ω = fundamental flutter frequency; and ω_f = fundamental free vibration frequency.
The aerodynamic parameter λ_o is dependent on R_x and is held fixed at its critical value which characterizes the flutter speed. A uniform reference panel with the same fixed values for λ_o and R_x is used in the nondimensionalization. The frequency parameter α may be varied in seeking the minimum-weight panel. After denoting the bending moment t^3w'' by q, the problem can be stated as follows: find that thickness distribution $t(x)$ and fundamental frequency parameter α which minimize the mass ratio

$$\int_0^1 t(x)\,dx$$

subject to

$$w'' = q/t^3(x) \qquad (2a)$$

$$q'' = -R_x q/t^3(x) - \lambda_o w' + (\alpha\pi)^4 t(x)w \qquad (2b)$$

with boundary conditions $w(0) = w(1) = q(0) = q(1) = 0$.

Now define the 2-vector $\mathbf{y}^T = (w, q)$ so that Eq. (2) can be written as

$$\mathbf{y}'' = \mathbf{f}[\mathbf{y}(x), t(x), \alpha] \qquad (3)$$

Let the dimensionless panel length $[0, 1]$ be divided into N equal intervals $[x_i, x_{i+1}]$ where each space point $x_i = ih$, $i = 0, 1, 2, \ldots, N$, where $h = 1/N$. Then adopt the notation $\mathbf{y}_i = \mathbf{y}(x_i)$, $t_i = t(x_i)$, and $\mathbf{f}_i = \mathbf{f}(\mathbf{y}_i, t_i, \alpha)$. The trajectory points $\mathbf{y}_i$, thickness ratio points t_i, and α are the discrete problem variables. Equation (3) is now approximated by the following set of difference equations:

$$\begin{aligned}
\mathbf{r}_1 &= 2\mathbf{y}_1 - \mathbf{y}_2 + h^2\mathbf{f}_1 = \mathbf{0}\\
\mathbf{r}_2 &= -\mathbf{y}_1 + 2\mathbf{y}_2 - \mathbf{y}_3 + h^2(\mathbf{f}_1 + 10\mathbf{f}_2 + \mathbf{f}_3)/12 = \mathbf{0}\\
&\vdots\\
\mathbf{r}_{N-2} &= -\mathbf{y}_{N-3} + 2\mathbf{y}_{N-2} - \mathbf{y}_{N-1} + h^2(\mathbf{f}_{N-3} + 10\mathbf{f}_{N-2} + \mathbf{f}_{N-1})/12 = \mathbf{0}\\
\mathbf{r}_{N-1} &= -\mathbf{y}_{N-2} + 2\mathbf{y}_{N-1} + h^2\mathbf{f}_{N-1} = \mathbf{0}
\end{aligned} \qquad (4)$$

The boundary conditions are always satisfied since $\mathbf{y}_0$ and $\mathbf{y}_N$ are equated to zero. Since Eq. (2b) contains a w' term, which is not compatible with Eq. (3), the finite difference $(w_{i+1} - w_{i-1})/(2h)$ has been used for $w'(x_i)$ in Eq. (4). The optimal thickness ratio is symmetrical about midchord and is zero at each end point. Note that Eq. (4) does not require the right-hand sides of Eq. (2) to be evaluated at either end point.

After approximating the mass ratio using the trapezoidal rule,

$$MR = 2h \sum_{i=1}^{(N-1)/2} t_i \text{ for odd } N \qquad (5)$$

the equality constraints (4) are adjoined to the mass ratio with a penalty function to form a sequence of unconstrained minimization problems of the form: minimize $F = MR + \frac{1}{2}K\mathbf{R}^T\mathbf{R}$, where $\mathbf{R}^T = (\mathbf{r}_1{}^T, \ldots, \mathbf{r}_{N-1}{}^T)$ and K is a positive penalty constant. Because of the large number of discrete variables, a conjugate gradient method has been used to solve each unconstrained problem. These subproblems are terminated whenever the square of the magnitude of the gradient vector for F is less than a preassigned tolerance $EPSI$ at the end of a complete conjugate gradient cycle. The sequence of subproblems is terminated whenever $\mathbf{R}^T\mathbf{R}$ is sufficiently small.

The convergence history for an $N = 33$ approximation with $R_x = 0$ and $\lambda_o = 341.0$ (see Ref. 4) is shown in Table 1. Note that this approximate solution ibdicates a weight savings of nearly

Table 1 Convergence history for $N=33$ approximation with $R_x=0$

K	$EPSI$	Cycles	$\frac{1}{2}\mathbf{R}^T\mathbf{R}$	MR	α
10,000	0.05	191	$0.26(10^{-5})$	0.8140	1.837
40,000	0.02	148	$0.33(10^{-6})$	0.8482	1.808
160,000	0.04	421	$0.43(10^{-7})$	0.8652	1.830
640,000	0.07	330	$0.72(10^{-8})$	0.8723	1.851

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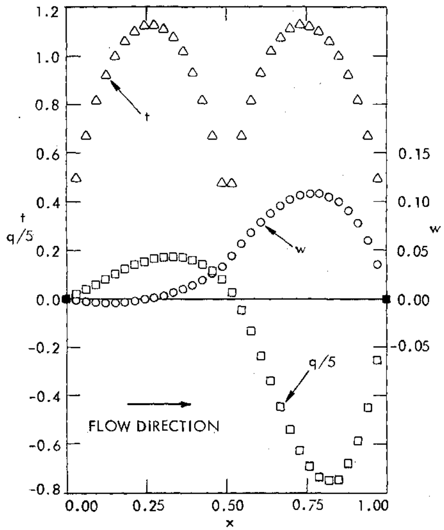

Fig. 1 Optimal thickness ratio, deflection, and bending moment for $R_x = 0$ and $N = 33$.

13% over the uniform reference panel. The solution for $K = 640{,}000$ is presented in Fig. 1. The symmetrical optimal thickness distribution in Fig. 1 reaches a maximum near $x = 0.26$ and $x = 0.74$ and becomes small at midchord. All solutions obtained during this study exhibit the same basic features as those shown in Fig. 1.

Approximate solutions with $N = 11$ were obtained for the case of constant inplane tensile stress ($R_x/\pi^2 = -2, -4$) and for the case of constant inplane compressive stress ($R_x/\pi^2 = +2, +3$). The results are presented in Table 2 along with comparable results for $R_x = 0$. Note that as more tension is applied, greater weight reduction is possible. Also, the optimal thickness distribution shifts toward midchord for decreasing R_x. In the process of computing values of the fundamental flutter-frequency parameter α for the uniform reference panel, values of λ_o more accurate than those given in Ref. 4 were obtained and adopted here. Note from Table 2 that the optimal α remains relatively close to the uniform reference value. The amount of computational effort required increases substantially with increasing λ_o.

Table 2 A comparison of $N=11$ solutions for various constant inplane stresses

R_x/π^2	K	$\frac{1}{2}\mathbf{R}^T\mathbf{R}$	λ_o	MR	α Uniform panel	α Optimal panel
−4	10,240,000	$0.21(10^{-10})$	697.10	0.8516	2.255	2.309
−2	10,240,000	$0.13(10^{-10})$	512.65	0.8592	2.059	2.104
0	2,560,000	$0.23(10^{-10})$	343.36	0.8642	1.813	1.856
+2	640,000	$0.33(10^{-10})$	190.95	0.8661	1.443	1.458
+3	640,000	$0.21(10^{-10})$	121.83	0.8635	1.082	1.040

References

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